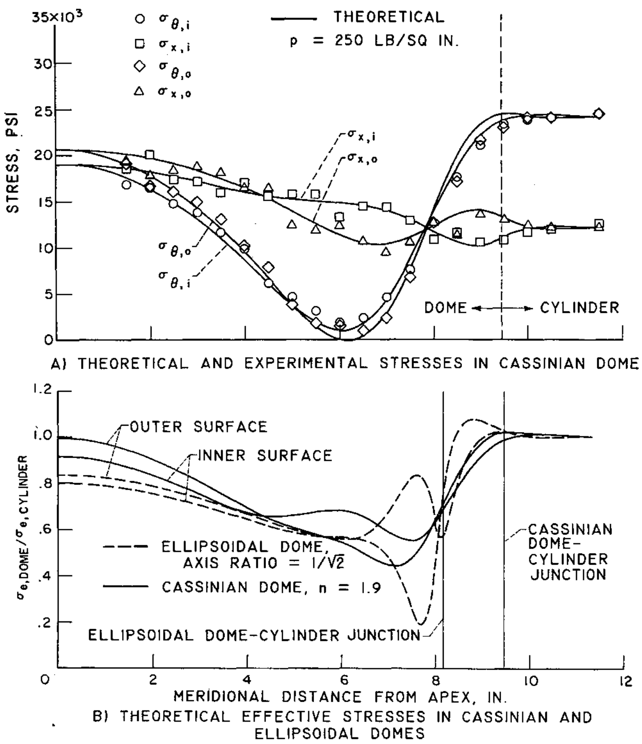

Fig. 4 Stresses in Cassinian and ellipsoidal domes

and experimental results. The theoretical stresses were computed using a numerical solution[6] of the Reissner equations[7] for the analysis of thin shells of revolution.

The Cassinian dome used for the experimental stress analysis had a parameter n of 1.90. This value was selected because it is the flattest head of this nature which has no compressive membrane stresses. Thus, circumferential buckling problems are eliminated. The flattest ellipsoidal dome that has no compressive membrane stress has a semiminor to semimajor axis ratio of 0.707. This ellipsoidal dome is compared geometrically with the Cassinian dome just described in Fig. 1d. It can be seen that the two curves are very similar in shape, although the depth of the Cassinian dome is larger than that of the ellipsoidal dome. However, because the Cassinian dome is nearly cylindrical over a large portion of its depth, interstage structures between propellant tanks with this type of dome probably would not extend from cylinder to cylinder. They would be more nearly the same length as those for tanks with ellipsoidal domes such as the one shown. Also, the volume of a propellant tank with Cassinian domes such as the one just described is slightly greater than the volume of a tank of equal length but having ellipsoidal domes with an axis ratio of 0.707.

The stresses were computed for a pressure vessel having the same outer radius and thickness as the Cassinian tank but with an ellipsoidal dome having an axis ratio of 0.707. Using the distortion energy theory, the effective stresses for this shell are compared to those for the shell with a Cassinian dome in Fig. 4b. The maximum effective stress ratio arising from the ellipsoidal dome is 1.076 vs 1.017 for the cylinder with the Cassinian dome. Thus, the yield pressure when using the Cassinian dome is about 6% higher than that obtained when using a comparable ellipsoidal dome. There is also considerably less bending in the knuckle region of the Cassinian dome, although the stresses near its apex are considerably higher but not critical.

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Performance of an Electromagnetic Actuation System

A. G. Buckingham*

Westinghouse Electric Corporation, Baltimore, Md.

Nomenclature

H_y	= component of the earth's field along the longitude line, oe
H_z	= component of the earth's field along the local geocentric vertical, oe
M_e	= magnetic moment of the earth, unit-pole/cm
θ_L	= latitude, deg
r	= distance from earth's center, cm
T	= torque, dyne-cm
B	= earth's field strength, gauss
I	= current amperes
N	= number of turns
A	= area of coil, cm²
θ	= angle between earth's field and coil field, deg
M_x, M_y, M_z	= components of momentum, dyne-cm-sec
I_{cx}, I_{cy}, I_{cz}	= coil currents, amp

I. Introduction

Attitude control systems may be classified into two general categories, depending upon the type of actuation system employed. One type uses a mass dispensing torque generation system such as compressed gas or chemical propellant, and a second type manipulates the natural forces of the space environment such as aerodynamic pressure, solar pressure, earth's mass attraction, and magnetically coupled forces. Mass dispensing systems have limited life due to fuel storability and stability problems, leakage, or the normal consumption of the fuel. The use of the majority of natural forces such as gravity gradient, solar pressure, and some forms of magnetic coupling provides very low torque levels; thus, mechanizations using these forces are limited. One method of magnetic coupling, however, provides comparatively high torque levels and a long life without restricting the attitude of the vehicle.

This method uses three mutually perpendicular coils for implementing momentum transfer and three mutually perpendicular inertia wheel controls for storing momentum and generating desired angular motions. Momentum transfer to the earth's field is accomplished by measuring the unwanted momentum stored in the inertia wheels with tachometer

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* Senior Engineer.

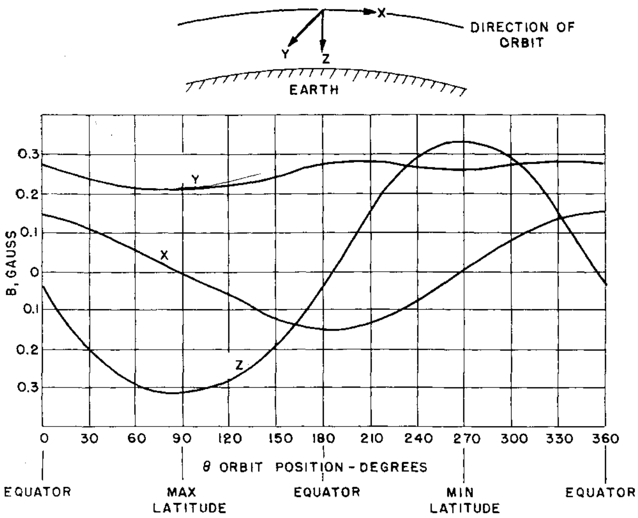

Fig. 1 Components of the earth's field as a function of orbit position at a 32° inclination

generators, measuring the components of the earth's field with a magnetometer, and computing in a simple analog manner the coil currents required to generate the desired momentum transfer torques.

This note presents the theory of operation, the mechanization, and the results of investigations made using the electromagnetic coupling method to perform attitude control.

II. Characteristics of the Earth's Field

As reported by Fischell[1] and others, the distribution of the earth's magnetic field generally can be described by the following equations:

$$H_x = (M_e/r^3)\cos\theta_L \tag{1}$$

$$H_z = (2M_e/r^3)\sin\theta_L \tag{2}$$

Using Vestine's data,[2] the components of the earth's field with respect to a vehicle that is oriented to the local vertical and the orbital plane were derived for an orbital inclination of 32° and an orbital altitude of 500 km. These are shown in Fig. 1 and can be simulated on an analog computer using sine and cosine functions. In the case of the Y axis, the superposition of a sine function on a d.c. level will accomplish the desired objective. The average of the vector sum of these components at both inclinations is similar to each other and amounts to approximately 0.346 gauss.

III. Principles of Operation

When the short-term effects of the earth's field (i.e., those occurring within an interval of several seconds) on a given space vehicle orbiting within a few thousand miles of the earth's surface are considered, the earth's field may be considered to be of constant magnitude and direction. If a current-carrying coil were placed in this field, a torque would act on the coil in accordance with the following expression:

$$T = (BINA/10)\sin\theta \tag{3}$$

The direction of the torque would be such as to cause the alignment of the earth's field with the coil field in a plane containing both the coil field vector and the earth's field vector. The action is similar to that of a d'Arsonval galvanometer. When three such coils are oriented mutually perpendicular to each other and the current to these coils is varied in a particular manner, the resultant coil flux may be directed in any direction and its magnitude varied over a wide range. The torque that would result by reacting the vector sum of the coil fields with the earth's field is given by the following expression:

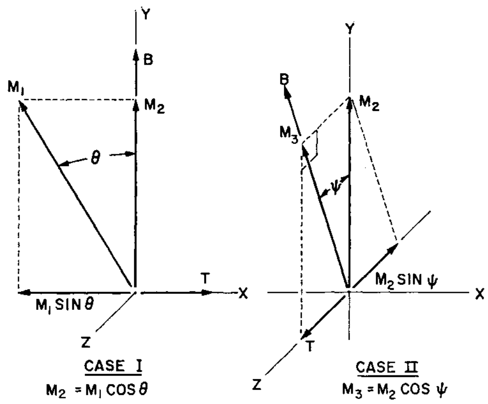

Fig. 2 Principle of momentum transfer

$$T = kB \times I \tag{4}$$

The direction of the torque is always in a plane perpendicular to the earth's field. By the proper computation of coil currents, the direction of the resultant torque may be varied to lie anywhere in the plane perpendicular to this field vector. The flux vector generated by the coil currents is not restricted to lie in a plane perpendicular to the earth's field vector, but optimum power efficiency results only if this condition is satisfied.

In order for a torquing system to be useful for a precise attitude orientation system, it must be capable of exerting torques in each of the three orthogonal control axes of the vehicle. The magnitude of the control torques must be greater than the sum of the internal and external disturbance torques at any instant of time. Using three orthogonal coils as a torquing system, the torque generated is always about an axis perpendicular to the earth's field and therefore can generate torques in only two of the vehicle's control axes when its third axis is parallel to this field. Hence an inertia wheel must be added to permit torque generation about the third axis. Further, when it is considered that the direction of the earth's field may be parallel to any axis of the vehicle in some missions, an inertial element must be added to each of three axes to assure complete control always.

Therefore, the control function about each of the three mutually perpendicular axes is accomplished by the wheel controls, with the function of the coil actuation system being to transfer or dump the angular momentum stored in the wheels to the earth. In essence, the coil actuation system provides an upper limit in angular velocity which a wheel system with fixed inertias will not exceed even though there is the constant presence of vehicle disturbing torques.

The operation of the momentum transfer or dumping system is depicted vectorially in Fig. 2. The total angular momentum of the vehicle which is stored in the three mutually perpendicularly mounted inertia wheels is represented initially by M_1 in case 1. When the momentum dumping system initially operates, the component of momentum $M_1 \sin\theta$ is removed, and the momentum that is left in the three wheels is represented by M_2 and is aligned with the earth's field vector. As the vehicle travels in orbit, the direction of the earth's field vector will change with respect to the momentum vector M_2, as shown in case 2 of Fig. 2. At this point, an additional amount of the remaining momentum ($M_2 \sin\phi$) stored in the inertia wheels is removed by the transfer system. The removal of wheel momentum continues by this process until the threshold of the momentum transfer is reached.

The ability to reduce continuously the total momentum stored in the vehicle depends upon the continuous change in direction of the earth's field vector with respect to the momentum vector of the vehicle. This continuous change in

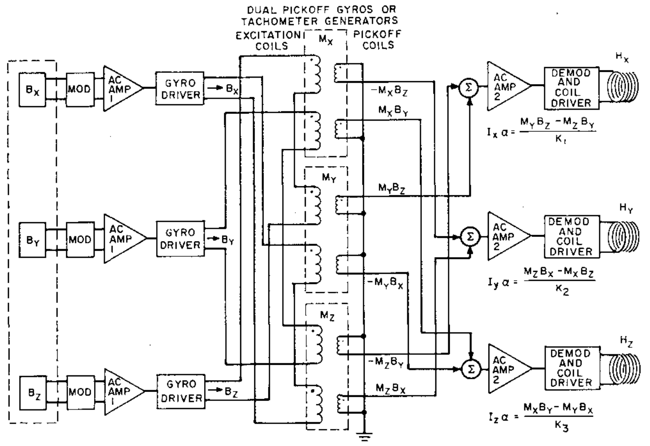

Fig. 3 Mechanization of momentum transfer system

the direction of the earth's field vector occurs for a celestially stabilized vehicle and also exists for all earth-oriented vehicles, including those which orbit in a plane near the equatorial plane.

IV. Mechanization

The three-axis mechanization of the momentum dumping system is shown in Fig. 3. The three orthogonal components of the earth's magnetic field are measured with a three-element magnetometer. The output of each magnetometer element energizes a pickoff coil on each of two rate gyros or tachometer generators which measure the vehicle rate or inertial wheel speed, respectively. Gyros are used where only a damping system is needed and will result in the angular rate of the vehicle being reduced to the threshold of the gyros. The tachometer generators are used when attitude or position control is desired. The outputs of the pickoffs are summed, amplified, and fed to each of the mutually perpendicular coils. The equations of the coil current which are solved by this mechanization are as follows:

$$I_{cx} = (M_yB_z - M_zB_y)/k_1 \quad (5)$$

$$I_{cy} = (M_zB_x - M_xB_z)/k_2 \quad (6)$$

$$I_{cz} = (M_xB_y - M_yB_x)/k_3 \quad (7)$$

These components currents are derived from the general vector equation given below with B^2 being constant for a given altitude:

$$I = (M \times B)/HB^2 \quad (8)$$

The denominator term HB^2 represents an automatic gain change term that need not be mechanized in the interest of system simplicity. The value of B^2 selected for the mechanization would be the average value of B^2 for the desired orbital inclination. The H term can be mechanized by performing step changes in gain that could occur if control is transferred from one level of inertia wheels to another. For example, if a particular attitude control system required a range of rate control of 10^5, then at least one step change in gain would be made by such a transfer.

V. Simulation Studies

The system shown in Fig. 3 and described by Eqs. (5–7) was simulated on a digital computer to determine the time to remove initial rates of 0.65 deg/sec about each of the control axes of a vehicle with moments of inertia of 800 slug-ft². The vehicle was oriented with respect to the celestial sphere at an orbital altitude of 500 miles. Approximately 30-w-hr† were required to reduce these initial rates to a level of 5 deg/hr in a period of 270 min. For the majority of orbits, the time required to remove a given amount of angular momentum may be reduced by increasing the torque capability of the coils. This can be accomplished by manipulating the equation $T = BINA/10$ such that the area, the turns, or the current to the coils is increased. In each application, the weight of the coil and the power consumed by it must be adjusted to meet a particular momentum transfer requirement.

† Coil wt = 20 lb but is not optimum design.

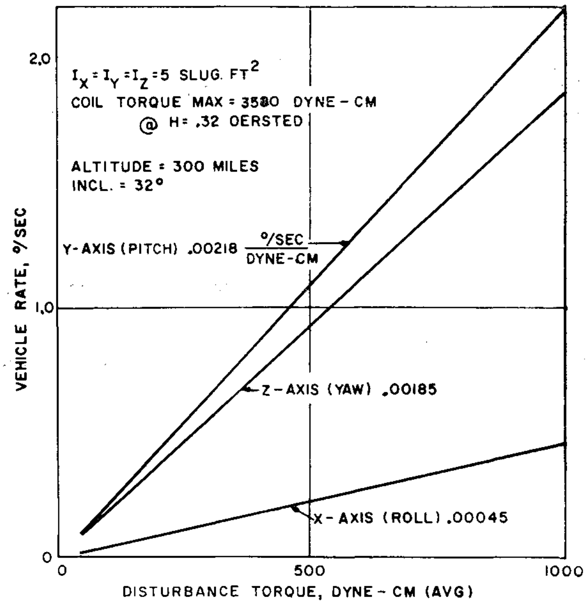

Fig. 4 Momentum storage requirements as a function of disturbance torque level

The system as shown in Fig. 3 also was simulated on an analog computer using an earth-oriented vehicle with inertias of 5 slug-ft² in each axis, an orbital inclination of 32°, and an orbital altitude of 500 km. A plot of the storage requirements of the wheels as a function of disturbance torque level is shown in Fig. 4. Referring to Figs. 1 and 4, it may be observed that, for a given disturbance torque level, the axis requiring the least momentum storage (x axis) depends upon the average field strength in the Y and Z axes. The average field strength in either of these axes is greater than the average field strength in the x axis.

VI. Comparison with Other Systems

A preliminary design of an electromagnetic actuation system was prepared for a typical cylindrical vehicle having two moments of inertia of approximately 40 slug-ft² and a roll moment of inertia of 5 slug-ft². It was compared with other types of actuation, including cold gas and chemical propellants. The use of gravity gradient torques and solar pressure was ruled out on the basis of the available torque levels and design complexities.

The function of the attitude control system in this application was to stabilize the vehicle during final stage burning in cases when the final stage was not spin-stabilized and to stabilize the vehicle's attitude for various length of time thereafter. In any of the cases considered, it was desirable to use electromagnetic actuation if the duration of the mission was greater than approximately one month, with power, weight, volume, and reliability being the prime factors in the selection of a particular system.

VII. Conclusions

For vehicles with an orbital altitude of less than approximately 40,000 miles and a life requirement of a month or more, the use of the electromagnetic actuation system consisting of coils and inertia wheels or only coils is both feasible and desirable.

The electromagnetic actuation system requires less volume and weight and has greater reliability than other means of actuation, such as mass dispensing with cold gas or bipropellants. It also compares favorably to the gravity gradient and solar pressure techniques when the overall attitude control system is considered.

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Structural Factors and Optimization of Space Vehicles

Frederick W. Ross*
General Dynamics Corporation, San Diego, Calif.

In the analysis of space vehicles, structural factors—originally defined as means of identifying structural mass—have been more recently treated as functions. This gives them a quite different connotation.

In 1947, Malina and Summerfield[1] defined the structural factor as the ratio of stage mass ejected, m_s, to stage mass with propellant, $\delta = m_s/(m_s + m_p)$, and, so defined, it pertains only to one particular stage, and involves neither the payload of that stage nor the payload or mass of any subsequent stages. Another factor alternately used is $r_s = m_s/(m_s + m_p + m_l)$, m_l being the stage payload; this factor includes all subsequent stages and their payloads.

Both factors are used by some authors,[2, 3] the choice depending upon the application, and both are convenient *realistic* terms used to identify particular structures. In analyses they are used as parameters, modifications in design being handled by usual methods of variable parameters or parametric curves.

However, in recent analyses,[4, 5] for the optimum number of stages, n_o, the optimum is assumed to be at $\partial\delta/\partial n$(or $\partial r_s/\partial n$) $= 0$, and δ (or r_s) is used as a function determined by n. With this usage δ (or r_s) is no longer an identifying parameter but a function describing a particular manner of variation of structural hardware mass with variation of n—e.g., $m_s = (\text{constant})(m_s + m_p)$.

To use the factor in this sense implies a foreknowledge of the functional influence of all the complex factors which determine m_s. Since it would be a monumental task to determine these precisely, it is pertinent to learn how sensitive important vehicle characteristics are to certain structural factors used as functions.

The system specific impulse, I_{ss}—introduced in a previous paper[3] as a measure for comparing propulsive systems that have wide differences in mass, and expressed in terms of δ by Eq. (11) of Ref. 3—for a single-stage vehicle becomes, with the aid of Eqs. (4) and (6) of the same reference,

$$I_{ss} = I_{sp}\{1 - [\log (1 + r_sG_s)]/\log G_s\} \qquad (1)$$

where $G_s = (m_s + m_p + m_{ls})/m_{ls}$. At $\partial I_{ss}/\partial G = 0$, where

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* John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division, San Diego, Calif.

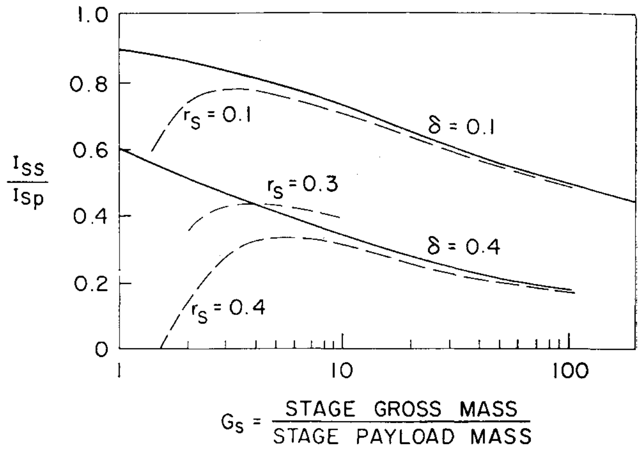

Fig. 1 Variation of I_{ss}/I_{sp} with different structural factors

$G_s = G_{so}$, $(I_{ss})_{\max} = 1/(1 + r_sG_{so})$. This depends only upon r_s and G_{so}, and is the same[3] as that for a multistage missile with proportionate staging.[1] Fig. 1 shows, after adjusting for $r_s = \delta(1 - G_s^{-1})$, that the essential difference introduced between the two curves obtained by treating r_s or δ as constant, enters only for $G < 3$.

For proportionate multistaging, I_{ss} can be given either by Eq. (24) of Ref. 3 in terms of δ and n, or by $I_{ss} = -I_{sp}(c/n)/\log\{e^{-c/n} - r_s\}$ in terms of r_s and n. These equations, and a similar equation in terms of two structural factors defined by Cooper[6] for $m_s = \epsilon m_o + fm_p$, where the first term is the "engine" and the second the "tankage" mass, are all of the form

$$I_{ss}/I_{sp} = -(c/n)/[\log (Ae^{-c/n} - B)] \qquad (2)$$

where the constants A and B are determined by the structural functions δ, r_s, or ϵ and f.

Similarly, the gross mass ratio m_o/m_l is given, respectively, by

$$G = (1 - \delta)^n(e^{-c/n} - \delta)^{-n} = (e^{-c/n} - r_s)^{-n} = [(1 + f)e^{-c/n} - (\epsilon + f)]^{-n}$$

where $c = v_r/GI_{sp}$, or in general by

$$G = R(e^{-c/n} - S)^{-n} \qquad (3)$$

where the quantities R and S are determined by δ, r_s, or ϵ and f.

Eq. (2) has a maximum, which for the form with δ is at $n \to \infty$ and for the other structural functions is at a finite n. Similarly, Eq. (3) has a minimum at n_m ranging from a small finite integer (5 in Fig. 2) to infinity, depending upon the choice of structural function. Because of this sensitivity to choice of structural function, the optimum n_o—i.e., the number of stages to give minimum G—is seen to be dependent more upon the selection of this function than upon the optimizing procedure, and to this extent is indeterminant.

Instead, if we observe that G decreases sharply from an infinite value where n is minimum, and levels off to a broad minimum (Fig. 2), regardless of the structural function used, we can define a preferred[3] minimum number of stages determined at the keen of the curve rather than the optimum at minimum G. In Fig. 2 this gives $n = 3$ or 4 for the preferred n rather than a range from $n = 5$ to infinity for the minimum G. In any practical application the small increase in G obtained by reducing n from 5 to 3 or 4 is more than offset by the reduction in complexity resulting from fewer stages. The preferred number, n_p, being independent of the choice of structural function, is then more closely related to practical design circumstances and is more definitive.

We can locate n_p at the knee of Eq. (3) from the curve graphically or from the integral portion of $n_p = 1 + (K - 1/\log S)c$. The term $-1/\log S$ is derived from the minimum